APICS Mathematics Contest 1991

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1. Find all pairs of real numbers (a, x) such that

$$(a-1)(|x|+|x+2|)=3a-4.$$

2. Prove that, for all natural numbers n, $2^{2n}+24n-10$ is divisible by $\mathbf{18}$.

3. Show that $\binom{\binom{n}{2}}{2} = 3\binom{n+1}{4}$.

4. Let $x_1, x_2, \ldots x_n$ be n real numbers in the closed interval $[0, 1]$. Show that there exists $x \in [0, 1]$ such that

$$\frac{1}{n}\sum_{i=1}^{n}|x-x_i| = \frac{1}{2}.$$

5. A contestant in a televised math contest has answered eight questions successfully and has a chance to win a car. The car is behind one of three doors, and she may pick one of these doors.

   At this point, it is the MC's custom to open an *empty* door which is not the one picked by the contestant. The MC then offers the contestant the opportunity to change her selection to the other closed door. If the car is behind the door that the contestant finally takes, she wins the car; otherwise she wins nothing.

   Should she stick to her first choice, switch, or doesn't it matter? Prove your answer.

6. Find the maximum value of $\tan(A)\cos(B)\sin(C)$, where $\triangle ABC$ is an acute-angled triangle in which $\angle A \le \angle B \le \angle C$.

7. Let ***AB,AC*** be equal chords of a circle. Find all chords that are divided into three equal pieces by the chords ***AB,AC***.

8. You are given an arbitrary sequence $u_1, u_2, \ldots, u_{2^n}$, where each of the u_i's is either **-1** or **+1**. You construct the new sequence $v_1, v_2, \ldots, v_{2^n}$ where $v_1 = u_1u_2, \ \ v_2 = u_2u_3, \ldots, \ \ v_{2^n-1} = u_{2^n-1}u_{2^n}$,

   and $v_{2^n} = u_{2^n}u_1$. Continue to construct new sequences successively, using the same rules. Show that after at most 2^n steps the resulting sequence will consist entirely of **1**'s.