2011 APICS Math Competition

Time: 3 hours.

Team members may collaborate with each other but not with others. Calculators and notes are forbidden.

Please write the answer to each question on a separate sheet (or sheets) of paper, and do not refer to other answers, as your answers to the various questions will be graded separately. Put your **team number** and the **question number** on ALL pages. Do not put your names, team name, or university on the answer sheets. Show all work.

Put your university, your own names, and your team number on the outside of the envelope before handing in your answers.

Few marks will be given for fragmentary or incomplete answers.

Each of the eight questions carries equal weight.

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QUESTIONS

1. Sketch: $\{(x, y) : \lfloor x^2 \rfloor + \lfloor y^2 \rfloor = 4\}$.

   For a real number x, $\lfloor x \rfloor$ is defined to be the greatest integer less than or equal to x.

2. Given a regular n-gon, how many of the convex quadrilaterals defined by quadruples of its vertices are non-rectangular trapezoids?

3. If α, β, γ are the roots of $x^3 - x - 1 = 0$, compute $\dfrac{1+\alpha}{1-\alpha} + \dfrac{1+\beta}{1-\beta} + \dfrac{1+\gamma}{1-\gamma}$.

4. Given a convex quadrilateral $ABCD$:

   (a) show that there always exist points $WXYZ$ such that Z is the midpoint of AW, W is the midpoint of BX, X is the midpoint of CY, and Y is the midpoint of DZ;

   (b) show that Area($\triangle ABW$) + Area($\triangle CDY$) = Area($\triangle BCX$) + Area($\triangle DAZ$).

5. The real valued infinitely differentiable function $f(x)$ is such that $f(0) = 1$, $f'(0) = 2$, and $f''(0)$=3. Furthermore, f has the property that $f^{(n)}(x)+f^{(n+1)}(x)+f^{(n+2)}(x)+f^{(n+3)}(x) = 0$ for all $n \geq 0$. Find $f(x)$.

6. Find the fraction with denominator less than 100 that is closest to (but not equal to) $\frac{3}{7}$.

7. If we remove the graph of the equation $x^4 + y^4 + z^4 = 2x^2y^2 + 2y^2z^2 + 2z^2x^2$ from R^3, how many connected components remain?

8. Suppose $\sin(a) + \sin(a + b) + \cos(b) = -1$. Show that $\cos(a) + \cos(a + b) + \sin(b) = 0$.