2008 APICS Math Competition

Time: 3 hours.

Team members may collaborate with each other but not with others. Calculators and notes are forbidden.

Please write the answer to each question on a separate sheet (or sheets) of paper, and do not refer to other answers, as your answers to the various questions will be graded separately. Put your **team number** and the **question number** on ALL pages. Do not put your names, team name, or university on the answer sheets. Show all work.

Put your university, your own names, and your team number on the outside of the envelope before handing in your answers.

Few marks will be given for fragmentary or incomplete answers.

This question booklet has 8 questions. Each of the eight questions carries equal weight.

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QUESTIONS

1. If I toss 4 dice, what is the probability that the product of the numbers is 36?

2. Find the minimum value of $f(x) = (1 + \cos(2x) + 8\sin^2(x))/\sin(2x)$ over the interval $(0, \pi/2)$.

3. What is the smallest 9-digit number (base 10) containing all the digits from 1 to 9 and divisible by 99?

4. Consider the 24 2 x 2 matrices which can be obtained by some arrangement of the four letters x,y,z,w. For a certain assignment of non-negative integers to x,y,z,w, we find that: 4 of these matrices have determinant 16; 4 have determinant -16; and 16 have determinant zero. Find all possible solution sets for $\{x, y, z, w\}$.

5. Let $f, g : \mathbb{N} \to \mathbb{N}$, where $\mathbb{N} = \{1, 2, 3, \ldots\}$ denotes the natural numbers. Suppose that f is surjective (onto) and that g is injective (1 - 1). Also, suppose that $f(n) \geq g(n)$ for all n. Prove that $f(n) = g(n)$ for all n.

6. For positive numbers $a_1, a_2, \ldots a_n$ and $b_1, b_2, \ldots b_n$, prove that
$$\sqrt[n]{(a_1 + b_1) \ldots (a_n + b_n)} \geq \sqrt[n]{a_1 \ldots a_n} + \sqrt[n]{b_1 \ldots b_n}.$$

7. When the Math Club advertises an "(M,N) sock hop", this means that the DJ has been instructed that the Mth dance after a fast dance must be a slow dance, while the Nth dance after a slow dance must be a fast dance. (All dances are slow or fast; the DJ avoids the embarrassing ones where nobody is quite sure what to do.) For some values of M and N this means that the dancing must end early and everybody can start in on the pizza; for other values the dancing can in principle go on forever. For which ordered pairs (M,N) is there no upper bound to the number of dances?

8. Quadrilateral ABCD is inscribed in circle Γ with $AD <$CD. Diagonals AC and BD intersect at E and M lies on EC so that $\angle CBM = \angle ACD$. Show that the circumcircle of $\triangle BME$ is tangent to Γ at B.