2006 APICS Math Competition

Time: 3 hours.

Team members may collaborate with each other but not with others. Calculators and notes are forbidden.

Please write the answer to each question on a separate sheet (or sheets) of paper, and do not refer to other answers, as your answers to the various questions will be graded separately. Put your **team number** and the **question number** on ALL pages. Do not put your names, team name, or university on the answer sheets. Show all work.

Put your university, your own names, and your team number on the outside of the envelope before handing in your answers.

Few marks will be given for fragmentary or incomplete answers.

This question booklet has 8 questions over 2 pages. Each of the eight questions carries equal weight.

QUESTIONS

1. The number 114438555 can be obtained as the product of four numbers, the difference of the greatest and least of which is 10. What are they?

2. Let P be a point inside a circle C. Find the locus of the centers of all circles ω which pass through P and are tangent to C.

3. For a rational number p/q (in lowest terms) define $f(p/q) = (p+1)/q$. Find the maximum value of $f(p/q)$ for $7/8 < p/q < 8/9$.

4. Three equal circles $\{C, D, E\}$ are often used in courses on elementary logic to form a *Venn diagram*, dividing the plane into eight nonempty regions, including the exterior, such that for each subset of $\{C, D, E\}$ there is a region which is contained in precisely those circles.

   Find a way to place four circles (which may be of different sizes) to create a sixteen-region Venn diagram, or show that this cannot be done.

5. Take a solid cylinder and split it lengthwise through the center. Place it on a plane, round side down. Tilt the plane. At what tilt angle of the plane does the half-cyclinder flop over to its flat side? Assume no slipping between the plane and the half-cylinder.

6. During my stay in Sydney, I am living at a bed and breakfast, where the owner has slightly eccentric rules about payment. She will not accept coins, cheques, debit or credit cards; give change; or accept bills of denomination \$50 or higher - and of course she requires me to settle my bill in full. I, in turn, am not prepared to pay more than I owe.

   Asuming that I have unlimited supplies of \$5, \$10 and \$20 bills, give (with proof) necessary and sufficient conditions for the number of different ways in which I can pay my bill to be a perfect square. (For instance, if I owe only \$25, I can pay in four ways: $20 + 5$, $10 + 10 + 5$, $10 + 5 + 5 + 5$, or $5 + 5 + 5 + 5 + 5$; and 4 is a perfect square.)

7. (a) Find the minimum positive value of $m = 36^k - 5^l$.

   (b) For the value of m found in part (a), find all solutions to $36^k - 5^l = m$.

8. Prove that

$$\cos(-89°) + \cos(-87°) + \cdots + \cos(87°) + \cos(89°) = \csc(1°)\ .$$