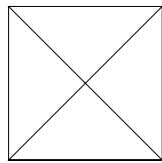


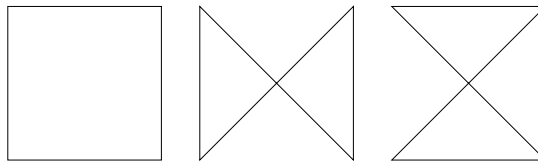
APICS Mathematics Competition 2002

October 18, 2002

1. For each positive integer n , define $s_n = (n + 13)(n + 77)$ and $t_n = n(n + 91)$. Let $S = s_1 + s_2 + \dots + s_{2002}$ and $T = t_1 + t_2 + \dots + t_{2002}$. Which is larger, S or T ? Prove your answer.
2. \mathbf{K}_n is a graph on n vertices, where every pair of vertices is joined by an edge. There are three different cycles of length 4 in the graph \mathbf{K}_4 .



\mathbf{K}_4

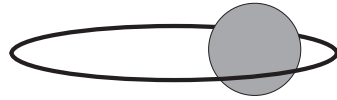


Cycles of length 4 in \mathbf{K}_4

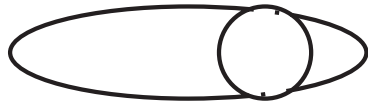
How many different cycles of length t are there in \mathbf{K}_n ?

3. Let $f(a, b)$ be the sum of all the positive integers between a and b inclusive. For example, $f(1, 5) = 1 + 2 + 3 + 4 + 5 = 15$.
 - a) Determine the value of $f(13, 53)$.
 - b) Determine the value of $f(13333\dots 33, 533\dots 3333)$, where there are n 3's in each expression.
4. Let P be a point in the plane with positive integer coordinates, and let O be the origin. Consider the circle with centre O , that passes through P . Let T be the tangent to the circle at P , and let T meet the x and y axes at X and Y , respectively. Prove that the area of triangle OXY cannot equal 2002.
5. For each positive integer n , let M_n be the square matrix where each diagonal entry is 2002, and every other entry is 1. Determine the smallest positive integer n for which $\det(M_n)$ is a perfect square.

6. An ellipse with formula $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a > b$) lies in the plane $z = 0$. What is the locus of the center of a sphere with radius b that moves so as to make contact with the ellipse at two points? (The sphere may be visualized as rolling on an elliptical ring that it is just small enough to pass through the centre of.)



Oblique view



Top view

7. At the Sackville Dim Sum restaurant, all dishes come in three sizes: small, medium, and large. Small dishes cost $\$x$, medium dishes cost $\$y$, and large dishes cost $\$z$, where x, y, z are positive integers with $x < y < z$. At this restaurant, there is no tax on any dish and the prices haven't changed for a long time.

Margaret, Art and Edgar had dinner there last night, and together, they ordered 9 small dishes, 6 medium dishes and 8 large dishes. When the bill came, the following conversation ensued:

Margaret: "This bill is exactly twice as much as when I last came here."

Art: "This bill is exactly three times as much as when I last come here."

Edgar: "Oh, that was a delicious meal, and very reasonably priced too. Even if we give the waiter a 10% tip, the total is still less than \$100."

Determine the values of x, y and z , and prove that your answer is unique.