2001 APICS Math Competition

Time : 3 hours

Team members may collaborate with each other but not with others. Calculators and notes are forbidden.

Please write the answer to each question on a separate sheet (or sheets) of paper, and do not refer to other answers, as your answers to the various questions will be graded separately. Put your team number and the question number ONLY on ALL pages. Show all work.

Few marks will be given for fragmentary or incomplete answers.

This question paper has 8 questions. Each of the eight questions carries equal weight.

1) P is a polynomial with integer coefficients. For 4 distinct integers, we have P(x) = 9. Show that there is no integers x with P(x) = 16.

2) Consider the Fibonnacci sequence (1, 1, 2, 3, 5, 8, …) with $a_{n+2} = a_{n+1} + a_n$ for all n. Show that $\sum_{n=1}^{\infty} \frac{a_n}{4^{n+1}}$ is equal to $\frac{1}{11}$.

3) Prove that among any 13 distinct real numbers, it is possible to find x and y such that $0 < \frac{x - y}{1 + xy} < 2 - \sqrt{3}$.

4) Show that $\lim_{n\to\infty} \frac{1}{n} \sqrt[n]{\frac{(2n)!}{n!}} = \frac{4}{e}$.

5) In ΔABC, R is the midpoint of BC. S is a point on AC such that CS=3SA. T is a point on AB such that area of ΔRST is twice the area of ΔTBR.
Find $\frac{AT}{TB}$.

6) Determine all functions f which are everywhere differentiable and satisfy
$f(x) + f(y) = f\left(\frac{x+y}{1-xy}\right)$ for all $x, y \in R$ with $xy \neq 1$.

7) Evaluate the integral $I(k) = \int_0^\infty \frac{\sin kx \cos^k x}{x} dx$, where $k \in N$.

Hint: recall that $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$

8) Find 3 consecutive integers, the first being a multiple of the square of a prime number, the second being a multiple of the cube of a prime number and the last being a multiple of the fourth power of a prime number.