1997 APICS MATHEMATICS CONTEST

Rules:

- Teams of two are to work in cooperation and to submit *one* set of answers each.
- Time: three hours.
- No notes, calculators, or other such aids are permitted.
- You may not communicate with noncontestants (except invigilators) or other teams.

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1. Find the smallest positive integer whose value is tripled if the left hand digit is transferred to the right side.

2. (a) Show that $\frac{\sin\theta}{\theta} = \cos\frac{\theta}{2}\cdot\cos\frac{\theta}{4}\cdot\cos\frac{\theta}{8}\cdot\ldots$. Assume that $\theta \neq 0$.

   (b) Show that

$$\frac{2}{\pi} = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2+\sqrt{2}}}{2}\cdot\frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2}\ldots.$$

3. What is the area of the floor region swept out when a bifold door (made up of two doors of width w, hinged at one end and with the other sliding in a linear track) is opened?

4. Two players alternately toss a coin. The winner is the first player who throws a head, followed immediately by a head thrown by the other player and a second head by the original player. What is the probability that the first player wins?

5. Find all solutions in complex numbers z,w to the system

$$z + w = 4$$
$$(z^2 + w^2)(z^3 + w^3) = 280.$$

6. Solve for x in terms of a,b,c if

$$\frac{1}{a-x}+\frac{1}{b-x}=\frac{1}{a-c}+\frac{1}{b-c}\ .$$

7. What is the smallest number of planes needed to divide a cube into congruent tetrahedra? Sketch your solution and show that it is the best possible.

8. You have n pennies and n-1 nickels arranged alternately; i.e., PNP...NP. A move consists of moving a pair of adjacent coins to one end of the row of coins and then sliding all the coins together. What's the minimum number of moves needed to get all the pennies at one end and all the nickels at the other?

9. Let n be a positive integer. Show that there always exists a positive integer m, whose digits are all 0's or 3's, which is divisible by n.