APICS Mathematics Contest 1993

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1. Evaluate the product

$$\left[1-\frac{1}{2^2}\right]\left[1-\frac{1}{3^2}\right]\cdots\left[1-\frac{1}{1993^2}\right].$$

2. Let $n > 1$ be a natural number. Prove that there exists an infinite number of solutions in positive integers to the equation

$$x^n + y^n = z^{n+1}\ .$$

3. Find the value of xyz given that

$$\begin{array}{ccccccc} x & + & y & + & z & = & 1 \\ x^2 & + & y^2 & + & z^2 & = & 2 \\ x^3 & + & y^3 & + & z^3 & = & 3 \end{array}$$

4. Let $a_1, a_2, \ldots, a_n$ be fixed real numbers. A function f is defined by

$$f(x) = a_1 \sin(x) + a_2 \sin(2x) + \ldots + a_n \sin(nx).$$

Suppose that for all x, we have $|f(x)| \le |\sin x|$.

Prove that $|a_1 + 2a_2 + 3a_3 + \ldots + na_n| \le 1$.

5.

S_0	S_1	S_2	S_3				S_{1993}	

A game consists of pushing a flat stone along the sequence of squares $S_0, S_1, S_2, \ldots, S_k, \ldots$ shown above. The stone is initially placed on square S_0 and is given a push to the right. When it stops on square S_k it is pushed again in the same direction. This continues until the stone either stops on square S_{1993} or goes beyond this square. Then the game stops. Successive pushes of the stone are independent of one another. Each time the stone is pushed, the probability that it will move exactly n squares is given by $1/2^n$. Determine the probability that the stone will stop exactly on square S_{1993}.

6. In triangle ABC, let D and F be the feet of the perpendiculars from A to C to their opposite sides BC and AB, respectively. Prove that triangle DBF is similar to triangle ABC.

7. Prove that the product of five consecutive positive integers can never be a perfect square.

8. You are given an unlimited supply of each of the digits **1,2,3** or **4**. Using only these four digits, you

construct n-digit numbers. Such an n-digit number will be called LEGITIMATE if it contains the digit **1** either an even number of times or not at all. Determine how many legitimate n-digit numbers there are.