APICS Mathematics Contest 1990

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1. Determine with proof whether $\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{m^2+n^2}$ is convergent or divergent.

2. Find a formula for $\sum_{k=1}^{k} k!k$ and prove it is valid.

3. Define a sequence A_n by

$$\begin{aligned} A_{n+1} &= 1 + A_n + \sqrt{1+4A_n} \\ A_0 &= 0. \end{aligned}$$

   Find with justification A_{1990}.

4.
   (a) Find with proof a pair of invertible **2**x**2** real matricies ***A*** and ***B*** such that all non-trivial linear combinations of ***A*** and ***B*** are also invertible.
   (b) Can you solve the problem of (a) for **3**x**3** maticies ***A***? Why(not)?

5. Show that the triangle formed by a tangent to a hyperbola and its two asymptotes has constant area.

6. Determine all real numbers ***m*** such that the equation

$$x^4 - (3m+2)x^2 + m^2 = 0$$

   has 4 real roots in arithmetic progression.

7. Let $a_1, a_2, \ldots a_n$ be ***n*** positive numbers and $a_{i_1}, a_{i_2}, \ldots a_{i_n}$ any permutation of them. Show that

$$\sum_{k=1}^{n}\frac{a_k^2}{a_{i_k}} \geq \sum_{k=1}^{n} a_k.$$

8. Suppose that a real valued function satisfies

$$|f(x) - f(y)| \leq |x-y|^p$$

   for some fixed $p > 1$ and all real ***x,y***. Show that ***f*** is a constant function.