APICS Mathematics Contest 1989

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1. Let b be an integer. Show that $8b + 1$ is a square if and only if b is of the form $b = r(r+1)/2$ for some integer r.

2. Find the nth derivative of the function:

$$f(x) = \frac{x^n}{1-x} \, .$$

3. Calculate the value of the expression:

$$\tan(20^\circ) \cdot \tan(40^\circ) \cdot \tan(60^\circ) \cdot \tan(80^\circ).$$

4. A set of $n(n+1)/2$ distinct values is randomly arranged in a triangle of n rows where the first row contains one value, the second row two values, and so on. Let M_k denote the largest value in the kth row. Find the probability that $M_1 < M_2 < \ldots < M_n$.

5. Determine all integer values of x and m for which

$$1x^3 + 9x^2 + 8x + 9 = m^3.$$

6. Let M be an $n \times n$ matrix of the form:

$$M = \begin{bmatrix} a_1 & a_2 & a_3 & \ldots & a_n \\ a_2 & a_2 & a_3 & \ldots & a_n \\ a_3 & a_3 & a_3 & \ldots & a_n \\ \ldots & \ldots & \ldots & \ldots & \\ a_n & a_n & a_n & \ldots & a_n \end{bmatrix} \, .$$

   Calculate the determinant of M.

7. Evaluate the expression [the exponents are all m]

$$\frac{1 + \frac{1}{2^m} + \frac{1}{3^m} + \frac{1}{4^m} + \ldots}{1 - \frac{1}{2^m} + \frac{1}{3^m} - \frac{1}{4^m} + \ldots}$$

   where $m > 1$ is a real number.

8. Suppose that n teams play in a round-robin tournament (i.e. a tournament in which each team plays each other team once). Each game ends in either a win or a loss. Show that there can be at most $2k + 1$ teams that have won exactly k games. Furthermore, if $m \geq 2k+1$, show that there can be exactly $2k + 1$ teams with k wins.