

APICS Mathematics Contest 1988

1. Show that $3a^4 - 4a^3b + b^4 \geq 0$ for all real numbers a, b .
2. In voting on a constitutional amendment, each province can either vote YES, vote NO or ABSTAIN. In order for the amendment to pass, 7 or more of the 10 provinces must vote YES, and at least one of Ontario and Quebec must be among the YES votes. How many different ways are there for the provinces to cast their votes so that the amendment is passed?
3. Show that $\sqrt{2}, \sqrt{3}, \sqrt{4}$ cannot all be terms of the same geometric sequence.
4. A lattice point is a point with integer coordinates. Show that if 9 lattice points, no 3 collinear, are taken in 3-dimensional space, then at least one of the segments joining pairs of these points must pass through another lattice point.
5. F and F' are the foci of an ellipse and P is any point on the ellipse. Prove that the circles described on $FP, F'P$ as diameters touch the circle which has the major axis of the ellipse as diameter.
6. For a nonempty subset of $\{1, 2, \dots, n\}$, multiply the elements of the subset together and then take the reciprocal of the product. When you sum these reciprocals over all nonempty subsets, what do you get? Prove your answer.
7. Let C_1, C_2, C_3 be circles which are each mutually tangent to the other two, and are each external to the other two. Let P_1, P_2, P_3 be their respective centres. Show that if $\angle P_1P_2P_3 = 90^\circ$, then the incircle of triangle $P_1P_2P_3$ has the same radius as C_2 .
8. A closed circular fence of radius 1 divides a large field into two distinct regions. A goat, free to graze in the outside region, is tied to the fence at a fixed point A . If the length of the rope is π , find the area which the goat can graze.