

APICS Mathematics Contest 1987

1. Find all different right-angled triangles with all sides of integral length whose areas equal their perimeters.
2. Sketch the graph and find the measure of the area bounded by the relation:

$$|X - 60| + |Y| = \left| \frac{X}{4} \right|.$$

3. Suppose $S(X)$ is given by

$$S(X) = X(1 + X^2(1 + X^3(1 + X^4(1 + \dots)))$$

Is $S(1/10)$ rational? (Explain your answer.)

4. Given a function f and a constant $c \neq 0$ such that

$f(x)$ is even

$g(x) = f(x - c)$ is odd

prove that f is periodic and determine its period.

5. Show that $\sqrt{3} \leq \exp\left(\int_{\pi/6}^{\pi/2} \frac{\sin x}{x} dx\right) \leq 3$.

6. Three equal circles each pass through the centres of the other two. What is the area of their common intersection.

7. Prove that $\frac{(1987^2)!}{(1987!)^{1988}}$ is an integer.

8. Find the sum of the infinite series

$$1 + 1/2 + 1/3 + 1/4 + 1/6 + 1/8 + 1/9 + 1/12 + \dots$$

where the terms are reciprocals of integers divisible only by the primes 2 or 3.