APICS Mathematics Contest 1985

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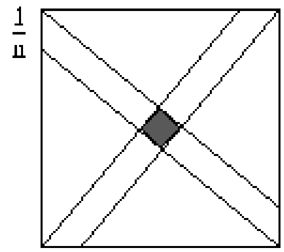
1. A unit square is dissected as shown measuring $1/n$ along each side. Show that the shaded quadrilateral is a square and find n so that its area is $1/1985$.



2. Evaluate the determinant of the $n \times n$ matrix

$$M = \begin{bmatrix} n & 1 & 1 & \dots & 1 \\ 1 & (n-1) & 1 & \dots & 1 \\ 1 & 1 & (n-2) & \dots & 1 \\ \vdots & \vdots & & \ddots & \vdots \\ 1 & 1 & \dots & \dots & 1 \end{bmatrix}$$

3. Define a sequence by $n(n-1)a_n = (n-1)(n-2)a_{n-1} - (n-3)a_{n-2} (n \geq 2)$ where $a_0 = a_1$ are any real numbers. Prove that $\sum_{n=0}^{\infty} a_n$ converges to $a_0 e$.

4. Find all positive integral solutions of

$$\binom{n}{k-1} - 2\binom{n}{k} + \binom{n}{k+1} = 0$$

where as usual $\binom{n}{k} = \frac{n(n-1)\dots(n-k+1)}{k(k-1)\dots 1}$.

5. Show that there is an integral $N > 0$ such that for all integral $n > N$ the equation $1985x + 68y = n$ has positive integral solutions (x, y). Find N.

6. Determine which of $\left(\frac{1}{2}e\right)^{\sqrt{3}}$ and $(\sqrt{2})^{\frac{1}{2}\pi}$ is greater.