APICS Mathematics Contest 1984

---

1. Does the series $\sum_{n=1}^{\infty} \frac{3n-2}{n^3+3n^2+2n}$ converge or diverge? If it converges, find the sum.

2. Determine all fourth degree polynomials $P(x)$ such that there is another polynomial $Q(x)$ with

$$P(x^2) - P(x) = (x^4 + x^3 + x^2 + x + 1)\ Q(x).$$

3. Solve for y_n given by

$$\begin{aligned} y_0 &= 5 \\ y_1 &= 9 \\ y_n &= n + 4y_{n-1} - 4y_{n-2} \quad (n \geq 2). \end{aligned}$$

4. Let T be the matrix $\begin{pmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ and $T^n = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$. Show that a_{11} and a_{22} are equal for each $n \geq 1$.

5. Is it possible to find two integers x and y such that the sum of their cubes is 6? Why not? What if x and y are allowed to be rational?

6. Let f be a twice differentiable function on $[0, \infty)$ such that $f(0) = 0$ and $f'(0) = 1$. If $f'(x) > 0$ and $f''(x) < 0$ for all $x \geq 0$, prove that $f(x)f(y) \leq f(xy)$ for all $x, y \geq 0$.