APICS Mathematics Contest 1981

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1. Let a_n be the general term of a sequence $a_1, a_2, a_3, \ldots a_n \ldots$, and let $S_n = \sum_{i=1}^{n} a_i$. Find, with justification, the term a_n if $\frac{Sm}{Sn} = \frac{m^2}{n^2}$, $n,\ m \in \{1, 2, 3, \ldots\}$.

2. Given positive real numbers ***a,b,c,d*** such that ***a* + *b* + *c* + *d* = 1**, find the largest possible value of $abcd^3$.

3. Let the plane be covered by a net of congruent squares and call the vertices of these squares ``lattice points''. Does there exist an equilateral triangle, all of whose vertices are lattice points? Explain.

4. Let D_n be the determinant of the $n \times n$ matrix

   $$\begin{vmatrix} 1 & 1 & 0 & . & . & . & . & . & 0 \\ -1 & 1 & 1 & 0 & & & & & 0 \\ 0 & -1 & 1 & 1 & & & & & 0 \\ . & & & & & & & & \\ . & & & & & & & & \\ . & & & & & 0 & -1 & 1 & 1 \\ . & & & & & & & & \\ 0 & & & & & & 0 & -1 & 1 \end{vmatrix}$$

   Assuming the limit exists, find $\lim_{n\to\infty} \left(\frac{D_n}{D_{n-1}}\right)$.

5. Let $A_1, \ldots A_n$ be non-collinear points in the plane and ***P*** and ***Q*** are points such that

   $$\sum_{i=1}^{n} \overline{A_i P} = \sum_{i=1}^{n} \overline{A_i Q} = S.$$

   Show that there exists a point ***K*** such that

   $$\sum_{i=1}^{n} \overline{A_i K} < S.$$

6. Let $f(n)$ denote the number of integers $r,\quad 0 \le r \le n$ such that $\binom{n}{r}$ is odd. Show that $f(n)$ is always a power of two.

7. Four flies sit at the corners of a square card table, side ***a***, facing inward. They start simultaneously walking at the same rate, each directing its motion steadily toward the fly on its right.

Find

(i) the equation of the path traced by one of the flies;
(ii) without calculus, the total distance travelled by each fly.