

APICS Mathematics Contest 1980

1. Show that

$$n^{1/n} < 1 + \left(\frac{2}{n}\right)^{1/2},$$

for all positive integers, n .

2. Which is the larger of the two numbers e^π and π^e ?

3. Show that the number x given by

$$x = 1 + \sqrt{2 + \sqrt{1 + \sqrt{2 + \dots}}}$$

is irrational.

4. How many $n \times n$ matrices, A , are there such that all the entries of A are either 0 or 1 , A^{-1} exists, and all the entries of A^{-1} are either 0 or 1 ?

5. Relate the positive solutions of the differential equation

$$\frac{d^2 y(x)}{dx^2} + f(x)y(x) = 0 \quad (a \leq x \leq b)$$

to those of the differential equation

$$\frac{dy(x)}{dx} + y^2(x) + f(x) = 0 \quad (a \leq x \leq b).$$

6. Let f be a non-negative function that is differentiable on the interval $0 \leq x \leq a$, where $a > 0$, and such that $f(0) = 0$, $f(a) = 1$, and

$$\int_0^a f(x) dx = 1.$$

Show that

$$\int_0^a \sqrt{\{f(x)\}^2 + \{f'(x)\}^2} dx \geq \sqrt{2},$$

if the integral exists.

7. Suppose that you stand on an infinite plane upon which are two circular towers of unequal radii. The apparent width of each tower is the angle subtended where you stand. Show that the locus of points from which both towers appear to be equally wide is a circle.