APICS Mathematics Contest 1979

1. Let $P(x)$ be any polynomial which satisfies the equation

$$P(x) = x^r + \frac{dP(x)}{dx}$$

for a fixed $r \geq 1$. Find $P(0)$.

2. Show that if n is an integer greater than 1, then $n^4 + 4$ is not prime.

3. Show that any convex polygon with area 1 can be covered by a parallelogram with area less than or equal to 2.

4. Let $S = \{x_1, x_2, \ldots, x_N\}$ be a set of real numbers. For each non-empty subset T of S, we form $\bar{x}_T =$ average of the elements of T. Find the median of the sequence $\{\bar{x}_T : T \subseteq S,\ T \neq \phi\}$. (The median of a sequence of numbers is the middle value when the numbers are arranged in non-decreasing order, e.g. the median of **2,4,7,9,9** is 7.)

5. Let S be a finite set consisting of n elements. Find the total number of unordered pairs of nonempty subsets (A, B) of S, such that $A \cap B \neq \phi$. (Unordered means that $(A, B) \equiv (B, A)$). Simplify the result as much as you can.

6. Prove the following inequality for all integral $n \geq 2$:

$$n^{\frac{1}{n-1}} > (n+1)^{\frac{1}{n}}.$$

7. Let $a_k, k = 0, 1, 2, \ldots$ be a sequence of non-negative real numbers such that

(i) $a_{m+n} \leq a_m \cdot a_n$ for all $m, n \geq 0$;
(ii) there exists an m_0 such that $a_{m_0} < 1$.

Show that $\sum_{k=0}^{\infty} a_k < \infty$, i.e. the $\lim_{N\to\infty} \sum_{k=0}^{N} a_k$ exists and is finite.